Math in Focus™

The Singapore Approach

Research Base

GREAT SOURCE®

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Introduction and Overview

***Math in Focus:* The Singapore Approach** is an innovative mathematics program for Grades K–5 that focuses on problem solving and the use of model drawing to drive the acquisition and application of mathematical skills. The principles underlying this program are drawn from a solid base of foundational research that has identified effective approaches to mathematics teaching.

This research base document cites the extensive literature on mathematics pedagogy and learning as it relates to the key features and philosophy of ***Math in Focus:* The Singapore Approach.**

A Brief History of *Math in Focus:* The Singapore Approach

Mathematics in Singapore is taught in an innovative way, with an emphasis on problem solving. This has not always been the case. Prior to 1980, the country imported all of its mathematics textbooks from other nations. Beginning in 1980, however, Singapore began to take a new approach to mathematics instruction. Instead of importing its mathematics textbooks, the Curriculum Development Institute of Singapore (CDIS) was established. One charge of CDIS was to develop primary and secondary textbooks. At the same time, the Ministry of Education, the centralized education authority in the country, set new goals for mathematics education. These goals emphasized a focus on problem solving and on heuristic model drawing. The CDIS incorporated these goals into the textbooks, and in 1982 the first Singapore math program, *Primary Mathematics 1–6*, was published. In 1992, a second edition was made available. The second edition revisions included an even stronger focus on problem solving and on using model drawing as a strategy to problem solve.

The country continued to develop its mathematics program. Further revisions included:

- Creating a tighter content focus of the mathematics curricula following a study to review the scope and sequence in 1998;
- Privatizing the production of the primary level mathematics textbooks in 2001, with the hope that collaboration among textbook publishers would lead to quality textbooks at more affordable prices; and
- Placing an even greater focus on developing mathematical concepts and fostering mathematical problem solving in 2006 revisions.

Did You Know...

***Math in Focus:* The Singapore Approach is the American version of *My Pals Are Here! Maths,* which is currently used in over 80% of schools in Singapore. It is the program that the 2007 TIMSS results reflect.**

The impact of Singapore's curricular and instructional initiatives is evidenced by the dramatic improvements in math proficiency for Singapore students on international assessments. In 1984, Singapore's students were placed 16th out of 26 nations in the Second International Science Study (SISS). By 1995, however, the Trends in International Mathematics and Science Study (TIMSS) ranked Singapore's students first among participating nations. The 2007 results also showed Singapore as a top-performing nation.

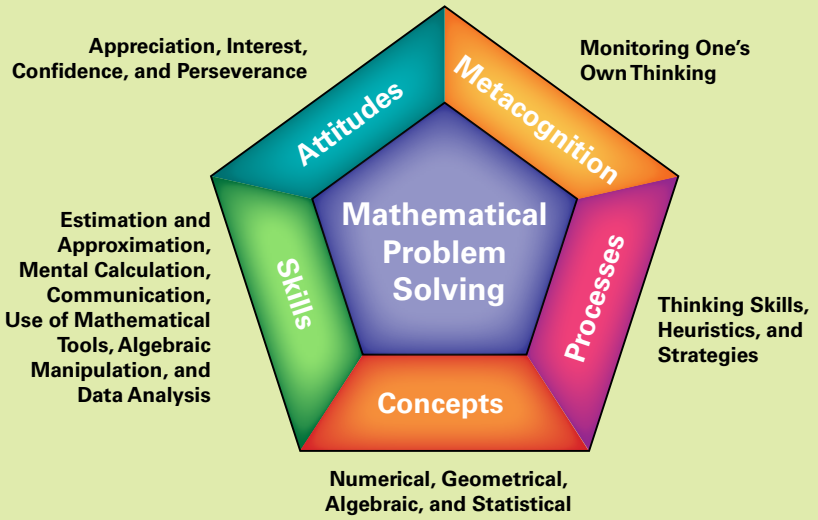
Why Singapore Math?

Data from the TIMSS has shown that the United States has been surpassed internationally in its mathematics performance (GONZALES ET AL., 2004). Meanwhile, Singapore's students have consistently been top performers in the TIMSS assessment. Clearly, Singapore math is effective.

The reason Singapore math works is simple—the program has a consistent and strong emphasis on problem solving. Other elements that contribute to the program's success include the program's focus on and support for building skills, concepts, and processes and its attention to developing students' metacognition and positive attitudes towards mathematics. Students are given opportunities to reflect on their thinking, communication, and problem solving so that they can apply these skills to varied problem solving activities. The Singapore Ministry of Education uses a pentagon with problem solving in the center to represent the elements of its program.

How Singapore math increases the emphasis on problem solving within its curricula is the key to its success. The program begins with a curriculum that includes fewer topics, but reaches greater depth at each level. The multi-page and multi-day lessons build students' knowledge systematically and thoroughly. Hands-On activities with manipulatives and extensive skills practice also contribute to student mastery. The use of visual representations builds students' skills *and* their conceptual understanding.

Singapore's Mathematics Framework



To engage all students, Singapore math uses minimal text and simple, direct visuals. As a result, all students, regardless of language skills, focus on the math lesson.

To allow all students to reach high levels of conceptual understanding and use of skills, a consistent approach of concrete to pictorial to abstract pedagogy is repeatedly employed. This use of scaffolding is found throughout the program. Students are given increasingly more intricate problems for which they draw on prior knowledge as well as recently acquired concepts and skills as they combine problem solving strategies with critical thinking skills.

In summary, Singapore math is successful because the program uses a focused, coherent syllabus that integrates concepts and skills in a concrete to pictorial to abstract way, all while emphasizing problem solving.

How Does This Program Connect to What Has Been Done Nationally?

In 2006, the National Council of Teachers of Mathematics (NCTM) expanded upon its earlier *Principles and Standards for School Mathematics* (2000) with a new way of looking at mathematics education—the *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*. The focal points describe the significant concepts and skills to be taught in mathematics at each grade level. According to NCTM (2006), "Organizing a curriculum around these described focal points, with a clear emphasis on the processes that *Principles and Standards* addresses in the Process Standards—communication, reasoning, representation, connections, and particularly, problem solving—can provide students with a connected, coherent, ever expanding body of mathematical knowledge and ways of thinking" (P.1).

***Math in Focus:* The Singapore Approach** aligns with the NCTM focal points and the NCTM standards. In a study by the American Institutes for Research that reviewed how the United States can learn from Singapore's mathematics system, the authors state that when "Compared with the NCTM framework…, Singapore's unique emphasis on multistep word problems, in particular, is consistent with its emphasis on promoting conceptual understanding through solving thoughtful problems" (GINSBURG, LEINWAND, ANSTROM, & POLLOCK, 2005, P. 35).

Furthermore, the NCTM (2006), the National Math Advisory Panel (2008), and the National Research Council (2001) have analyzed and synthesized the research base on U.S. performance in mathematics and generated these recommendations for mathematics instructional materials:

1. A focused, coherent curriculum, with increasing depth over time and without unnecessary repetition
2. A balanced approach to teaching concepts and skills
3. The use of concrete models and visual representations
4. An emphasis on problem solving, including complex problems with multiple steps and non-routine problems

***Math in Focus:* The Singapore Approach** was created specifically to meet these recommendations. It does so in these ways:

- The program addresses fewer topics in greater depth at each level.
- The program develops concepts and skills in tandem.
- The program uses clear and engaging visuals to present concepts and model solutions.
- The program uses a scaffolded approach to solving word problems and uses model drawing to build students' success and confidence.

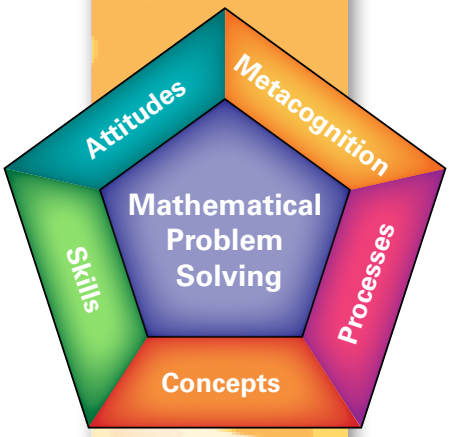
In addition to alignment with the work of key groups nationally, ***Math in Focus:* The Singapore Approach** was also developed after multiple rounds of focus group discussions with experienced educators in all regions. This extensive market research ensures that the program meets the current and specific needs of students and teachers around the country. For more details, visit **www.greatsource.com/mathinfocus.**

Strand 1: The Singapore Model

Singapore math uses a focused, coherent syllabus that integrates concepts and skills in a concrete to pictorial to abstract way, all while emphasizing increasingly rich and complex problems.

The Pentagon Framework

The learning of mathematics is founded in the student acquiring and applying a multitude of concepts and skills in order to solve a wide range of mathematical problems in varying non-routine and real-world situations. According to the Singapore Ministry of Education's Primary Mathematics Syllabus (2000), "The development of mathematical problem solving ability is dependent on five inter-related components, namely, Concepts, Skills, Processes, Attitudes and Metacognition" (P. 6). Thus, the Singapore Ministry of Education uses a graphic to represent their vision for mathematics teaching—a pentagon, with problem solving in the center and these five interdependent, necessary elements surrounding it.



PRINCIPLES FROM RESEARCH

"Singapore's framework lays out a balanced set of mathematics priorities centered on problem solving. It includes an emphasis on computation skills along with more conceptual and strategic thinking..."(GINSBURG ET AL., 2005, P. XI)

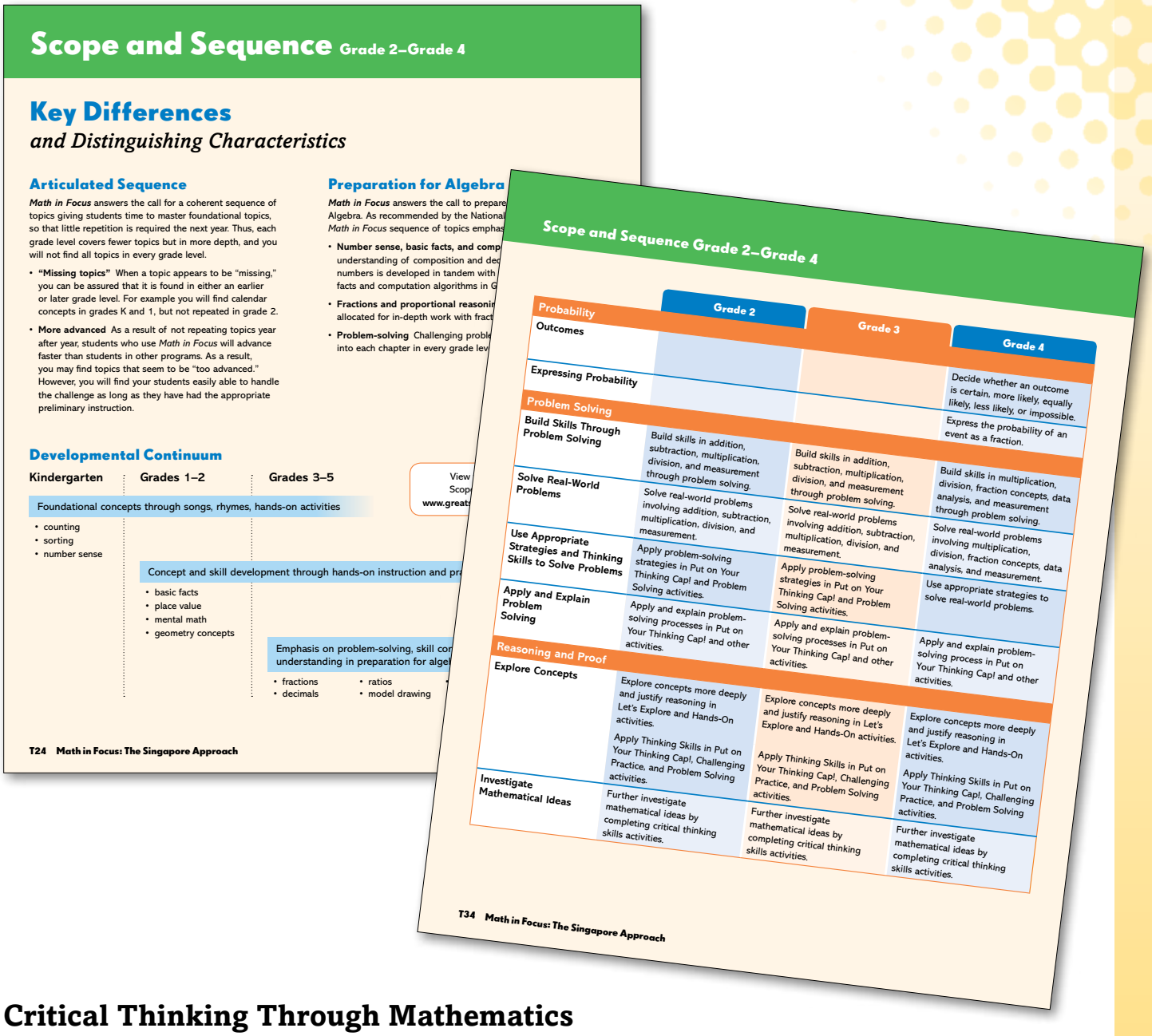
According to Lesh and Zawojewski (2007) "Neither concept development nor the development of problem solving abilities proceeds in the absence of the development of beliefs…and other components…A fresh view of problem solving needs to view the learning of mathematics and problem solving as integrated, as largely based on modeling activity…." (PP. 778–779).

TURNING PRINCIPLES INTO SUCCESS

Each time a new concept is introduced in ***Math in Focus:*** **The Singapore Approach**, problem solving is central. Students are taught specific problem solving strategies in a carefully sequenced manner of concrete to pictorial to abstract.

Each lesson in ***Math in Focus:*** **The Singapore Approach** develops students' problem solving abilities by building their conceptual understandings, skills, mathematical processes, attitudes towards math, and self-awareness.

- Concepts are developed by following a sequence of concrete to pictorial to abstract.
- Concepts and skills are reinforced through opportunities to practice and apply.
- Carefully sequenced and scaffolded lessons allow students to reach mastery—and build confidence—before moving on.
- Routine and non-routine problems require that students solve each problem with a conscious awareness of how to approach problem solving.
- **Guided Practice, On Your Own, Hands-On activity, Let's Explore!, Math Journal,** and **Put On Your Thinking Cap!** incorporate all components of the pentagon.



Scope and Sequence Grade 2–Grade 4

Key Differences
and Distinguishing Characteristics

Articulated Sequence

Math in Focus answers the call for a coherent sequence of topics giving students time to master foundational topics, so that little repetition is required the next year. Thus, each grade level covers fewer topics but in more depth, and you will not find all topics in every grade level.

- **"Missing topics"** When a topic appears to be "missing," you can be assured that it is found in either an earlier or later grade level. For example you will find calendar concepts in grades K and 1, but not repeated in grade 2.
- **More advanced** As a result of not repeating topics year after year, students who use *Math in Focus* will advance faster than students in other programs. As a result, you may find topics that seem to be "too advanced." However, you will find your students easily able to handle the challenge as long as they have had the appropriate preliminary instruction.

Preparation for Algebra

Math in Focus answers the call to prepare
Algebra. As recommended by the National
Math in Focus sequence of topics emphas

- **Number sense, basic facts, and comp** understanding of composition and dec numbers is developed in tandem with facts and computation algorithms in G
- **Fractions and proportional reasoni** allocated for in-depth work with fract
- **Problem-solving** Challenging proble into each chapter in every grade lev

Developmental Continuum

Kindergarten | Grades 1–2 | Grades 3–5

Foundational concepts through songs, rhymes, hands-on activities
- counting
- sorting
- number sense

Concept and skill development through hands-on instruction and pr
- basic facts
- place value
- mental math
- geometry concepts

Emphasis on problem-solving, skill cor understanding in preparation for algel
- fractions
- decimals
- ratios
- model drawing

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Scope and Sequence Grade 2–Grade 4

	Grade 2	Grade 3	Grade 4
Probability			
Outcomes			Decide whether an outcome is certain, more likely, equally likely, less likely, or impossible.
Expressing Probability			Express the probability of an event as a fraction.
Problem Solving			
Build Skills Through Problem Solving	Build skills in addition, subtraction, multiplication, division, and measurement through problem solving.	Build skills in addition, subtraction, multiplication, division, and measurement through problem solving.	Build skills in multiplication, division, fraction concepts, data analysis, and measurement through problem solving.
Solve Real-World Problems	Solve real-world problems involving addition, subtraction, multiplication, division, and measurement.	Solve real-world problems involving addition, subtraction, multiplication, division, and measurement.	Solve real-world problems involving multiplication, division, fraction concepts, data analysis, and measurement.
Use Appropriate Strategies and Thinking Skills to Solve Problems	Apply problem-solving strategies in Put on Your Thinking Cap! and Problem Solving activities.	Apply problem-solving strategies in Put on Your Thinking Cap! and Problem Solving activities.	Use appropriate strategies to solve real-world problems.
Apply and Explain Problem Solving	Apply and explain problem-solving processes in Put on Your Thinking Cap! and other activities.	Apply and explain problem-solving processes in Put on Your Thinking Cap! and other activities.	Apply and explain problem-solving process in Put on Your Thinking Cap! and other activities.
Reasoning and Proof			
Explore Concepts	Explore concepts more deeply and justify reasoning in Let's Explore and Hands-On activities. Apply Thinking Skills in Put on Your Thinking Cap!, Challenging Practice, and Problem Solving activities.	Explore concepts more deeply and justify reasoning in Let's Explore and Hands-On activities. Apply Thinking Skills in Put on Your Thinking Cap!, Challenging Practice, and Problem Solving activities.	Explore concepts more deeply and justify reasoning in Let's Explore and Hands-On activities. Apply Thinking Skills in Put on Your Thinking Cap!, Challenging Practice, and Problem Solving activities.
Investigate Mathematical Ideas	Further investigate mathematical ideas by completing critical thinking skills activities.	Further investigate mathematical ideas by completing critical thinking skills activities.	Further investigate mathematical ideas by completing critical thinking skills activities.

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Critical Thinking Through Mathematics

The ability to think critically is essential in our 21st century world. ***Math in Focus:*** **The Singapore Approach** develops students' abilities to think critically by providing opportunities for students to carry out investigative activities and discuss alternate solutions to open-ended routine and non-routine problems. In addition, the journaling component provides students with the opportunity to reflect on the mathematics.

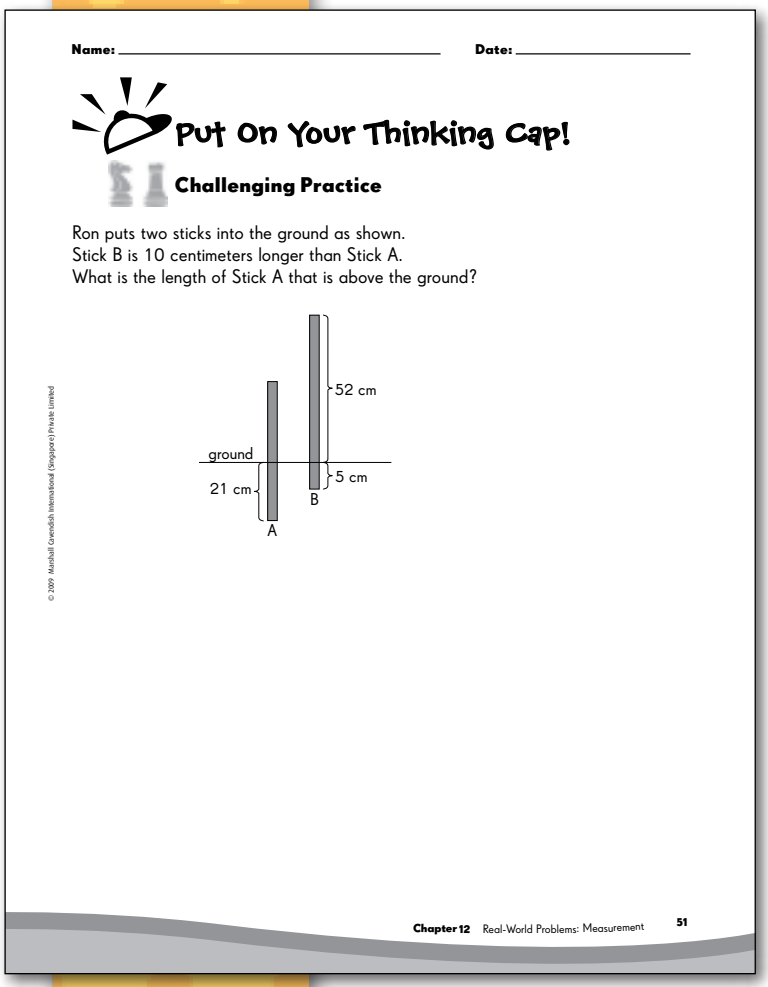
PRINCIPLES FROM RESEARCH

Mathematics is one means toward the desired end of critical thinking; to think mathematically is to think critically. "The student is responsible for figuring out how to solve the problem as well as finding the solution. It is the strategies, rather than the answers, that are the site of the mathematical thinking, and it is these strategies that reveal the assumptions a student is making about how mathematics works" (FENNEMA, CARPENTER, & LAMON, 1991, P. 231).

Weak calculation skills are not incompatible with higher-level understandings. As a result, it is essential that all students are observed engaging in mathematical activities across a broad range of complexity. As Franke, Kazemi, and Battey (2007) note: "(A)ll students need opportunities to develop both concepts and skills, to develop flexibility in their abilities to engage with mathematical ideas, and to engage in what some may call higher order or critical thinking" (P. 231).

TURNING PRINCIPLES INTO SUCCESS

Put On Your Thinking Cap!, at the end of each chapter, promotes critical thinking and typically focuses on problem solving strategies. Each ***Math in Focus:*** **The Singapore Approach** chapter concludes with Put On Your Thinking Cap!, which challenges students to solve non-routine questions.



Name: _____ Date: _____

Put On Your Thinking Cap!

Challenging Practice

Ron puts two sticks into the ground as shown.
Stick B is 10 centimeters longer than Stick A.
What is the length of Stick A that is above the ground?

These problems ask children to draw on deep prior knowledge as well as recently acquired concepts, combining problem solving strategies with critical thinking skills that include:

- Classifying
- Comparing
- Sequencing
- Analyzing parts and whole
- Identifying patterns and relationships
- Induction (from specific to general)
- Deduction (from general to specific)
- Spatial visualization

Differentiated Instruction

Differentiated instruction targets learners at all ways and levels of understanding, from English Language Learners, to struggling learners, to gifted learners.

PRINCIPLES FROM RESEARCH

The theory of differentiated instruction rests on the belief that teachers should vary and adapt instruction to meet individual, diverse student needs in their classrooms. Researchers support the power of differentiation:

"Differentiated instruction is an instructional process that has excellent potential to positively impact learning by offering teachers a means to provide instruction to a range of students in today's classroom situations" (HALL, STRANGMAN, & MEYER, 2003).

Stetson, Stetson, and Anderson (N.D.) report the results of a study of 48 elementary teachers using differentiated instruction and found that teachers utilizing such instruction techniques reported increases in student motivation and a greater sense that students' needs were being met. Students in classrooms in which the teacher used differentiated instruction also took greater ownership of their own learning and had greater academic success.

Research increasingly demonstrates that the same math instructional strategies that have been shown to be effective with native English speakers are also effective with English Language Learners with an emphasis on communication and problem solving possibly being more effective in teaching math to English Language Learners (SEE LEIVA, 2007).

TURNING PRINCIPLES INTO SUCCESS

Math in Focus: **The Singapore Approach** provides for mathematical instruction at a variety of levels to target learners at all levels of understanding, from struggling learners to gifted learners.

FOR STRUGGLING LEARNERS: **Reteach** pages provide more exposure to concepts for those students who need more time to master new skills or concepts. Additionally, the ***Math in Focus:*** **The Singapore Approach** Teacher's Editions provide tips for helping struggling students at point of use.

FOR ON-LEVEL STUDENTS: **Extra Practice** pages correlate directly to the Workbook practices. Here again, Put On Your Thinking Cap! questions provide more practice on both non-routine and strategy-based questions.

FOR ADVANCED STUDENTS: **Enrichment** exercises of varying complexity provide advanced students opportunities to extend the concepts, skills, and strategies they have learned in the Student Book and Workbook.

The clear drawings and visual aspect of ***Math in Focus:*** **The Singapore Approach** means the entire program is inherently accessible to English Language Learners. Additionally, the ***Math in Focus:*** **The Singapore Approach** Teacher's Editions provide lesson-specific suggestions for facilitating instruction for English Language Learners.

The Three Clusters: Learning, Consolidating, and Applying Concepts

The ***Math in Focus:*** **The Singapore Approach** materials purposefully and consistently follow an instructional pathway of:

- Learning concepts and skills through visual lessons and explicit instruction;
- Consolidating concepts and skills with practice, activities, and journaling; and
- Applying concepts and skills with extensive problem solving practice.

PRINCIPLES FROM RESEARCH

According to Romberg (1992), "Learning does not occur by way of passive absorption. Instead, individuals approach each new task with prior knowledge. They assimilate new information and construct their own meanings…. As instruction proceeds, they continue to use these routines to solve problems, despite their new knowledge of more formal procedures; they will accept new ideas only when it is no longer feasible to use prior routines"(P.31).

"[L]earning is said to have occurred when students develop an understanding of the relationships between and among concepts and between concepts and procedures" (STEIN, REMILLARD, & SMITH, 2007. P. 331).

TURNING PRINCIPLES INTO SUCCESS

LEARNING concepts and skills is all about understanding the how and the why. Therefore, each lesson in the Student Book is introduced with a Learn element. Mathematical concepts are presented in a straightforward visual format, with specific and structured learning tasks.

CONSOLIDATING concepts and skills leads to a deeper understanding of math. Opportunities to develop deeper understanding are provided through extensive practice in the lesson using Guided Practice and Let's Practice, hands-on work in pairs and small groups with the Hands-On activity and Let's Explore, and communication and reflection with Let's Explore and Math Journal.

APPLYING concepts and skills builds real-world problem solvers. ***Math in Focus:*** **The Singapore Approach** embeds problem solving throughout each lesson. This allows students to have frequent exposure and frequent practice with problems that encompass previous skills and concepts in word problems that grow in complexity.

Strand 2: Visualization/Representation

Researchers have concluded that visualization is a powerful problem solving tool and can be helpful in all kinds of mathematical problems, not only geometric problems (VAN GARDEREN, 2006). According to NCTM (2000), "Instructional programs from prekindergarten through grade 12 should enable all students to—

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena".

Moving from Concrete, to Pictorial, to Abstract Representation

According to Miller and Hudson (2007), "A systematic way to integrate the use of manipulative devices and pictorial representations into explicit instruction designed to teach important concepts is through use of the concrete-representation-abstract (CRA) teaching sequence" (P. 49). In ***Math in Focus:*** **The Singapore Approach**, first concrete manipulatives and then pictorial illustrations are used as visual aids to give students the tools to formulate, represent, and reason through different types of complex problems. Once students understand the concrete and the pictorial abstract concepts, then the abstract representation using numerals and mathematical notation is used.

PRINCIPLES FROM RESEARCH

"Many students who have difficulty grasping abstract mathematical concepts would benefit from visual representations of mathematical ideas. As part of this approach, the Singapore [math approach] illustrations demonstrate how to graphically decompose, represent, and solve complicated multi-step problems" (GINSBURG, ET AL., 2005, P. XII).

"It was found that the Singapore textbook included features that can help pupils acquire strong foundations in mathematics as well as develop good habits of mind…The strong foundation is achieved through extensive use of pictorial representations, especially in the early part of the development of a skill or concept…" (HAR, 2005).

"This concrete to pictorial to abstract approach benefits all students but has been shown to be particularly effective with students who have mathematics difficulties, mainly because it moves gradually from actual objects through pictures and then to symbols" (JORDAN, MILLER, & MERCER, 1998).

TURNING PRINCIPLES INTO SUCCESS

Within the lessons of the ***Math in Focus:*** **The Singapore Approach** series, instruction consistently follows the sequence of moving students from concrete to pictorial to abstract.

- **Concrete:** Manipulatives are used to explain abstract mathematical concepts.
- **Pictorial:** Pictures, models, and diagrams are used to present examples with solutions.
- **Abstract:** Only numerals, mathematical notation, and symbols are used once students are familiar with the abstract representation.

Teaching Model Drawing

When learning with a model approach, students create diagrams to represent problems and concepts with bars. Drawing these types of models helps students to visualize strategies for problem solving and to make algebraic concepts more concrete. Model drawing can

- help children solve simple and complex word problems.
- develop algebraic thinking.
- help students visualize the part-whole structure of the problem.
- develop students' operational sense.
- foster proportional reasoning.

PRINCIPLES FROM RESEARCH

According to De Corte, Verschaffel, and Greer (2000), modeling makes the connection between the real and the abstract in mathematics: "The process of modeling constitutes the bridge between mathematics as a set of tools for describing aspects of the real world, on the one hand, and mathematics as the analysis of abstract structures, on the other; as such it is a pervasive aspect of mathematics…. [and a] powerful way to connect pupils' mathematics problem solving to the real world..."(P. 71).

"There is increasing recognition that…imagery based processes play an important role in all levels of mathematical problem solving…" (CAMPBELL, COLLIS, & WATSON, 1996, P. 177).

One effective form of modeling in mathematics is bar modeling. "Bar modeling is a specific variant of the common Draw a Picture mathematics problem solving strategy. Because Singapore Math uses this one variant consistently, students know what kind of picture to draw. That's an advantage if the bar model is versatile enough to apply to many complex problems—and it is. It is especially useful for problems that involve comparisons, part-whole calculations, ratios, proportions, and rates of change. It communicates graphically and instantly the information that the learner already knows, and it shows the student how to use that information to solve the problem" (HOVEN & GARELICK, 2007, P. 28).

TURNING PRINCIPLES INTO SUCCESS

Math in Focus: **The Singapore Approach** uses bar models as a problem-solving tool that is taught explicitly in Grades 2–5. Students become familiar with this systematic way to translate complex word problems into mathematical equations, and avoid the common issue of not knowing where to start.

Using Multi Models

Students are given the opportunity to experience multiple models of the same concept through the concrete to pictorial to abstract approach applied throughout ***Math in Focus:*** **The Singapore Approach**.

PRINCIPLES FROM RESEARCH

According to research, the context of learning is important. Learners taught in only one context and with only one model may not be able to transfer this knowledge and apply it to new situations. According to the National Research Council (2000), "When material is taught in multiple contexts, people are more likely to extract the relevant features of the concepts and develop a more flexible representation of knowledge that can be used more generally" (P. 236). Further, Llinares, and Roig (2008) have suggested that "[mathematical] learning can be identified as the extent to which learners are able to use the model constructed in different situations" (P. 507).

"By using multiple embodiments to represent a concept, students begin to recognize the general, abstract concepts that the various embodiments are intended to convey. Similarly, by creating, adapting, and comparing several structurally similar embodiments of a mathematical model, students have opportunities to compare and contrast models, to think about the similarities and differences among them, and to investigate the relationships among alternative models. The generalizing of knowledge from the initial situation to an unfamiliar, yet somewhat close situation is what constitutes transfer from a models-and-modeling perspective" (LESH & ZAWOJEWSKI, 2007).

"Providing various mathematical representations acknowledges the uniqueness of students and provides multiple paths for making ideas meaningful. Engendering opportunities for students to present mathematical solutions in multiple ways (e.g., symbols, graphs, tables, and words) is one tool for cognitive development…" (OJOSE, 2008, P. 28).

TURNING PRINCIPLES INTO SUCCESS

The daily lessons in ***Math in Focus:*** **The Singapore Approach** present new concepts and skills using multiple models to further deepen students' understanding. Let's Explore! then provides opportunities for students to connect these new skills with prior concepts, carry out investigative activities, and discuss alternate solutions to open-ended questions. This allows students to see and use multiple approaches to addressing the problem being presented.

Each ***Math in Focus:*** **The Singapore Approach** chapter concludes with Put On Your Thinking Cap! which challenges students to solve more challenging non-routine questions and apply skills and concepts in different ways.

Scaffolding

One way to adapt instruction to the needs of individual learners is through scaffolding, "the systematic sequencing of prompted content, materials, tasks, and teacher and peer support to optimize learning" (DICKSON, CHARD, & SIMMONS, 1993, P. 12). The ultimate goal of scaffolding is to gradually remove the supports as the learner masters the task. In the research, "scaffolding has

repeatedly been identified as one of the most effective instructional techniques available" (GRAVES & AVERY, 1997, P 138). ***Math in Focus:*** **The Singapore Approach** provides scaffolding through its step-by-step, individualized instruction.

PRINCIPLES FROM RESEARCH

"The most basic strategy for supporting students' expectations of success (and their related perceptions and beliefs, such as a sense of efficacy) involves two basic elements. The first is to design for success by assigning tasks on which students can succeed if they invest reasonable effort. The second is to provide whatever scaffolding may be needed to help students acquire and apply concepts, skills, and abilities as they work on assignments. This strategy involves building on students' current knowledge, which in turn requires understanding what they already know and where they are headed" (NATIONAL RESEARCH COUNCIL, 2001, PP. 339–340).

Various types of instructional strategies can provide needed scaffolding: "Teachers and other competent individuals can provide scaffolding by providing models or different representations, by thinking aloud, by giving hints, by providing useful feedback, and by guiding students through the first parts of tasks. These types of scaffolding would be expected to support and develop positive self-efficacy judgments as the student comes to accomplish tasks that he or she would not be able to accomplish alone" (SCHUNK, PINTRICH, & MEECE, 2008, P. 329).

"Scaffolded instructional discourse provides for negotiation of meaning and transfer of responsibility.... Negotiation is reflected both in teachers' attempts to build understanding with their students and to help them attain higher levels of competence. As students demonstrate increasing competencies, teachers withdraw assistance and they transfer responsibility for learning to the student. Transfer of responsibility increases student ownership while it holds students accountable for their learning" (TURNER, ET AL., 2002, P. 90).

TURNING PRINCIPLES INTO SUCCESS

Math in Focus: **The Singapore Approach** uses a scaffolded approach to introducing new concepts and increasingly difficult problems. Scaffolding is apparent in the concrete to pictorial to abstract approach that appears throughout the program, and in the sequencing of the word problems that go from one-step to two-step to multi-step.

Math in Focus: **The Singapore Approach** scaffolds student learning, teaching skills and concepts to mastery, so students can take accountability for their own learning and solve increasingly complex, non-routine problems. Instruction begins with Let's Practice which highlights students' prerequisite skills and leads to On Your Own independent practice then builds these concepts through Let's Explore where students investigate alternate solutions to open-ended problems, and culminates with Put On Your Thinking Cap!, challenging students to solve non-routine problems.

Strand 3: Conceptual Understanding

According to Hiebert and Grouws (2007), "If conceptual understanding… is a valued learning goal, then students will need opportunities to develop such understanding. (C)onceptual understanding grows as mental connections become richer and more widespread" (P. 387).

Moving from Concrete, to Pictorial, to Abstract Understanding

"For students to understand such mathematical formalisms, we must help them connect these formalisms with other forms of knowledge, including everyday experience, concrete examples, and visual representations. Such connections form a conceptual framework that holds mathematical knowledge together and facilitates its retrieval and application" (DONOVAN & BRANSFORD, 2005, P. 364).

PRINCIPLES FROM RESEARCH

Numerous studies demonstrate the positive effects of using concrete materials and pictorial representations when instructing students with learning disabilities, dyslexia, and other language difficulties. Such research shows that such approaches successfully help students with learning disabilities master math concepts including algebra skills, basic math facts, coin sums, fractions, multiplication, and place value (SEE MILLER & HUDSON, 2007).

One way to use concrete materials and pictorial representations in explicit instruction is the CRA (concrete-representation-abstract) approach. The teacher first illustrates the concept using manipulative devices. Representational pictures or tallies are then substituted for the manipulatives. Finally, instruction proceeds to the abstract level of problem solving without objects or representations.

"When students connect manipulative models to their intuitive, informal understanding of concepts and to abstract symbols, when they learn to translate between representations, and when they reflect on the constraints of the manipulatives that embody the principles of a mathematics system…they build Integrated-Concrete ideas"(CLEMENTS, 1999, PP. 55–56).

TURNING PRINCIPLES INTO SUCCESS

Manipulative use plays a key role in the concrete to pictorial to abstract sequence and classroom manipulative kits are available to assist with this. Manipulatives such as coin and bill combination sets, connecting "snap" cubes, counters in multiple colors, counting tape, craft sticks, and a demonstration clock are just a sample of the items found in the kits to allow for concrete representations of concepts and skills.

Math in Focus: **The Singapore Approach** consistently employs the Concrete > Pictorial > Abstract pedagogy.

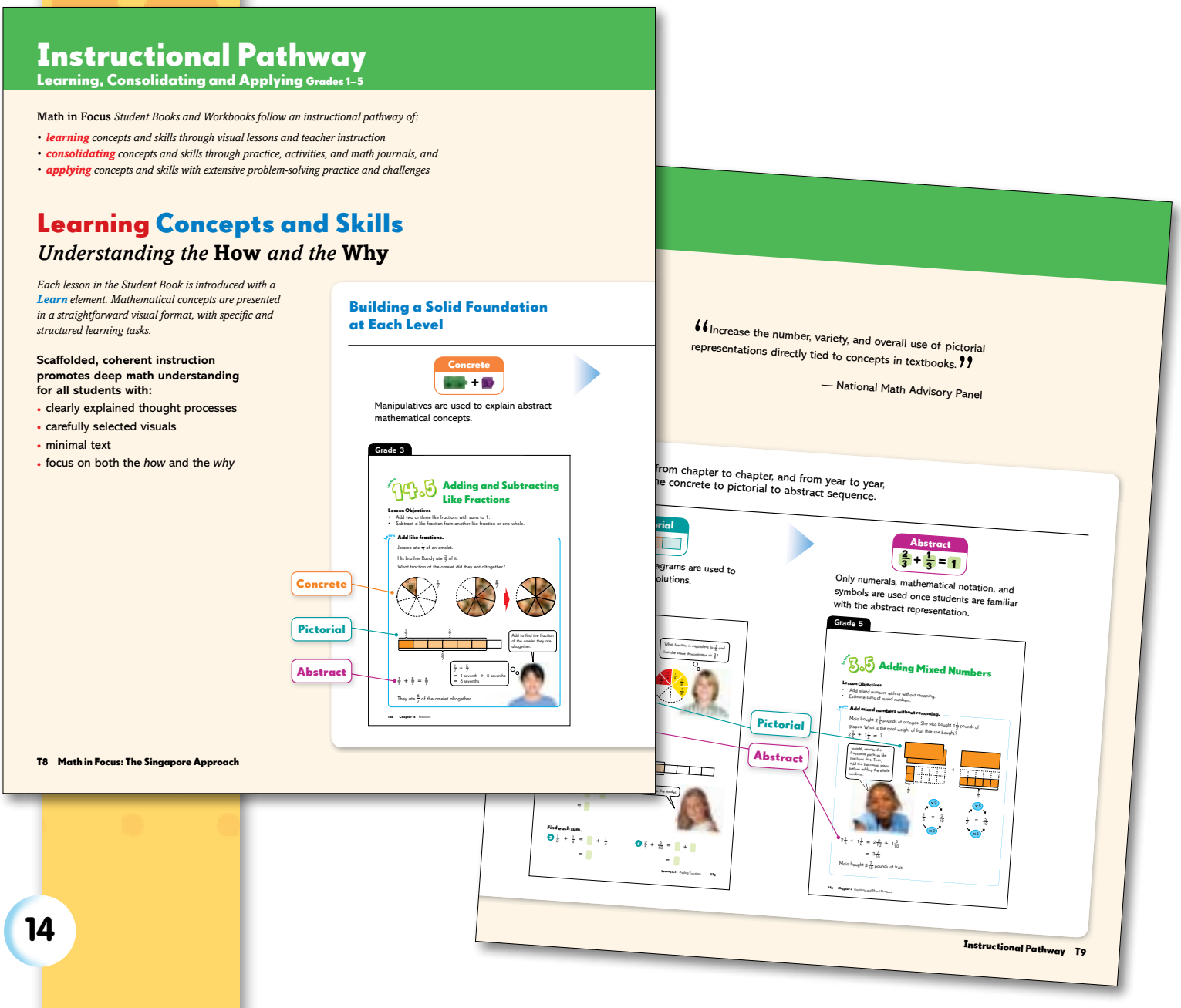
- **Concrete** manipulatives are used to explain abstract mathematical concepts.
- **Pictures,** visual models, and diagrams are used to present examples with solution.
- Numerals, mathematical notation, and symbols are used once students are familiar with the **abstract** representation.

Teaching Concepts and Skills/Building Mathematical Understanding

Facility with mathematical concepts and skills is essential to success in mathematics. In a review of research, Hiebert and Grouws (2007) determined that "instruction emphasizing conceptual development facilitated skill learning as well as conceptual understanding" (P. 387).

PRINCIPLES FROM RESEARCH

"Learning the 'basics' is important; however, students who memorize facts or procedures without understanding often are not sure when or how to use what they know. In contrast, conceptual understanding enables students to deal with novel problems and settings. They can solve problems that they have not encountered before" (NATIONAL COUNCIL OF MATHEMATICS, 2000, "THE LEARNING PRINCIPLE").



"Developing mathematical understanding requires that students have the opportunity to present problem solutions, make conjectures, talk about a variety of mathematical representations, explain their solution processes, prove why solutions work, and make explicit generalizations" (FRANKE. ET AL., 2007).

"Knowledge that is taught in only a single context is less likely to support flexible transfer than knowledge that is taught in multiple contexts. With multiple contexts, students are more likely to abstract the relevant features of concepts and develop a more flexible representation of knowledge" (BRANSFORD, ET AL., 2000, P. 78).

TURNING PRINCIPLES INTO SUCCESS

Math in Focus: **The Singapore Approach** provides opportunities in each chapter for students to develop deeper understanding through exploration, discussion, and reflection. Participation in the Hands-On activities, Games, Let's Explore!, Put On Your Thinking Cap!, and Math Journal all encourage concept and skill development as well as present extensive practice with problem solving and reasoning.

Multi-page and multi-day lessons provide the support and time needed to allow students to gain a deep understanding of concepts through multiple approaches. Each chapter in ***Math in Focus:*** **The Singapore Approach** uses multiple approaches to introduce and build on new concepts. For example, in Kindergarten students may use cubes to count and compare in the Learn activity, and then use cutouts in the Hands-On activity to further develop the count and compare concept.

Using Routine and Non-Routine Problems

"The development of strategies for solving nonroutine problems depends on understanding the quantities involved in the problems and their relationships as well as on fluency in solving routine problems. Similarly, developing competence in solving nonroutine problems provides a context and motivation for learning to solve routine problems and for understanding concepts such *as given, unknown, condition,* and *solution*" (NATIONAL RESEARCH COUNCIL, 2001, P. 127).

PRINCIPLES FROM RESEARCH

According to Bransford, Brown, and Cocking (2000) it is important for a research-based approach to problem solving to affirm the value of problem-based learning by requiring "students to work through extensive problem sets that include routine and nonroutine applications in a wide variety of real-world contexts" (P. 78).

"A fundamental characteristic needed throughout the problem solving process is flexibility. Flexibility develops through the broadening of knowledge required for solving nonroutine problems rather than just routine problems…(N)onroutine problems are problems for which the learner does not immediately know a usable solution method. Nonroutine problems require productive thinking because the learner needs to invent a way to understand and solve the problem" (NATIONAL RESEARCH COUNCIL, 2001, P. 126).

"There is a need for teachers to present non-routine and open-ended problems that will develop students in the use of thinking skills and heuristics" (HO, TEONG, & HEDBERG, N.D.).

TURNING PRINCIPLES INTO SUCCESS

***Math in Focus:* The Singapore Approach** is based on the belief that applying concepts and skills builds real-world problem solvers and, to accomplish this, the program embeds problem solving throughout each lesson.

Word problems grow in complexity from one-step to two-step to multi-step, which enhances the students' ability to think critically in a systematic process. Students learn to use model-drawing strategies to represent problems, which provides students with the skills and strategies necessary to approach both routine and non-routine problems.

Challenging, non-routine problems are presented in Put On Your Thinking Cap! To solve these problems, students must draw on both prior knowledge and recently acquired skills and concepts as well as think critically about how to best combine problem solving strategies.

Teaching Procedural Fluency: The Role of Language

In its standard on communication, the NCTM (2000) states that, "Communication is an essential part of mathematics and mathematics education. …Communication can support students' learning of new mathematical concepts as they act out a situation, draw, use objects, give verbal accounts and explanations, use diagrams, write, and use mathematical symbols" (PP. 59–60).

PRINCIPLES FROM RESEARCH

Numerous studies have emphasized the importance of communication in the mathematics classroom. For example, Lovitt and Curtis (1968) found that encouraging a student to verbalize problems before giving a written response increased the rate of correct answers. Gersten and Chard (2001) note that "encouraging students to verbalize their current understandings and providing feedback to the student increases learning."

In *How Students Learn: Mathematics in the Classroom,* Donovan and Bransford state, "Classroom communication about students' mathematical thinking greatly facilitates both teacher and student assessment of learning" (P. 239).

"Mathematical conversations provide opportunities for teachers to hear regularly from their students and to learn about the range of ideas students have about a particular mathematical idea, the details supporting students' ideas, the values students attach to those ideas, and the language students use to express those ideas. The knowledge teachers gain from engaging with their students in conversations is essential for teaching for understanding" (FRANK, ET AL., 2007, P. 237).

TURNING PRINCIPLES INTO SUCCESS

***Math in Focus:* The Singapore Approach** provides opportunities for students to communicate and reflect on the math they are practicing. By working in pairs or small groups through the Hands-On activities or Let's Explore!, students are able to discuss their thinking and their reasoning and therefore further develop their understanding of the concepts being presented. Also, the Math Journal provides students with a chance to write and reflect on their mathematical learning. The Teacher's Edition includes Common Error alerts to help teachers identify and correct likely misconceptions.

Using Ongoing Assessment

Monitoring students' learning is an essential part of planning effective instruction that will meet the specific needs of all students. Assessment opportunities in ***Math in Focus:*** **The Singapore Approach** offer a complete picture of student progress. The Student Book, the Workbook, and the Assessments book all work in concert to provide both short-term and long-term assessment options.

PRINCIPLES FROM RESEARCH

Ongoing assessment is essential in the classroom. First, assessments "permit the teacher to grasp students' preconceptions, which is critical to working with and building on those notions. Once the knowledge to be learned is well defined, assessment is required to monitor student progress (in mastering concepts as well as factual information), to understand where students are in the developmental path from informal to formal thinking, and to design instruction that is responsive to student progress" (DONOVAN & BRADFORD, 2005, P. 16).

The National Research Council's (1999), *Testing, Teaching, and Learning,* which focuses on recommendations for Title I students, recommends that "Teachers should administer assessments frequently and regularly in classrooms for the purpose of monitoring individual students' performance and adapting instruction to improve their performance" (P. 47).

As Baker, Gersten, and Lee (2002) report "…One consistent finding is that providing teachers and students with specific information on how each student is performing seems to enhance mathematics achievement consistently" (P.67). Similarly, the National Mathematics Advisory Panel (2008) points out "…teachers' regular use of formative assessment improves students' learning" (P. XXIII).

TURNING PRINCIPLES INTO SUCCESS

Math in Focus: **The Singapore Approach** provides assessment at each stage of students' learning:

- A process for assessing prior knowledge at the beginning of each lesson through **Quick Check** and **Pretest**
- A process for ongoing diagnostic evaluations to adjust instruction if needed with **Guided Practice** and **Common Errors** alerts
- A formal assessment to determine mastery for a review or a final grade by using **Chapter Review/Test, Chapter Assessment, Cumulative and Mid-Year Assessments, Benchmark Tests,** and **Mid-Year** and **End-of-Year Tests.**

Math in Focus: **The Singapore Approach** makes it easy for teachers to adapt instruction through the use of ongoing diagnostic assessments such as

- **Guided Practice** in the Student Book: After each Learn element students work out Guided Practice examples with either peer or teacher input. Tips in the Teacher's Edition help in assessing student understanding.
- **Common Errors** in the Teacher's Edition: Common Error alerts help teachers recognize and correct potential misconceptions before students practice on their own.

Strand 4: Problem Solving

Problem solving is a fundamental part of mathematics—and everyday life. The ability to solve problems is both a goal of mathematics—and a tool within mathematics. As such, problem solving should be integrated into all mathematical learning situations (NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS, 2000).

Teaching Deep Processing of Information, Reasoning, and Thinking

Opportunities are provided throughout the ***Math in Focus:*** **The Singapore Approach** program for students to use reasoning skills to make connections between prior knowledge and newly presented material. These connections will help to foster a deeper understanding of the math concepts.

PRINCIPLES FROM RESEARCH

A concrete to abstract sequence of learning helps students to attain a deep level of mathematical thinking: "If students can see the nature of the problems that mathematical conventions were designed to solve, their conceptions of what mathematics is can be influenced productively" (DONOVAN & BRANSFORD, 2005, P. 574).

According to Stein, Remillard, and Smith (2007), "(Students) having the opportunity to work on challenging tasks in a supportive classroom environment translated into substantial learning gains on an instrument specially designed to measure high level thinking and reasoning processes" (P. 358).

TURNING PRINCIPLES INTO SUCCESS

Math in Focus: **The Singapore Approach** follows a scaffolded path where visual lessons and teacher instructions allow for students to acquire deep learning and mastery of concepts and skills before they move on. Let's Practice, Hands-On activities, and Math Journals allow students to consolidate concepts and skills. Extensive problem solving and challenges such as Let's Explore! and Put On Your Thinking Cap! require students to apply concepts and skills in new, challenging contexts.

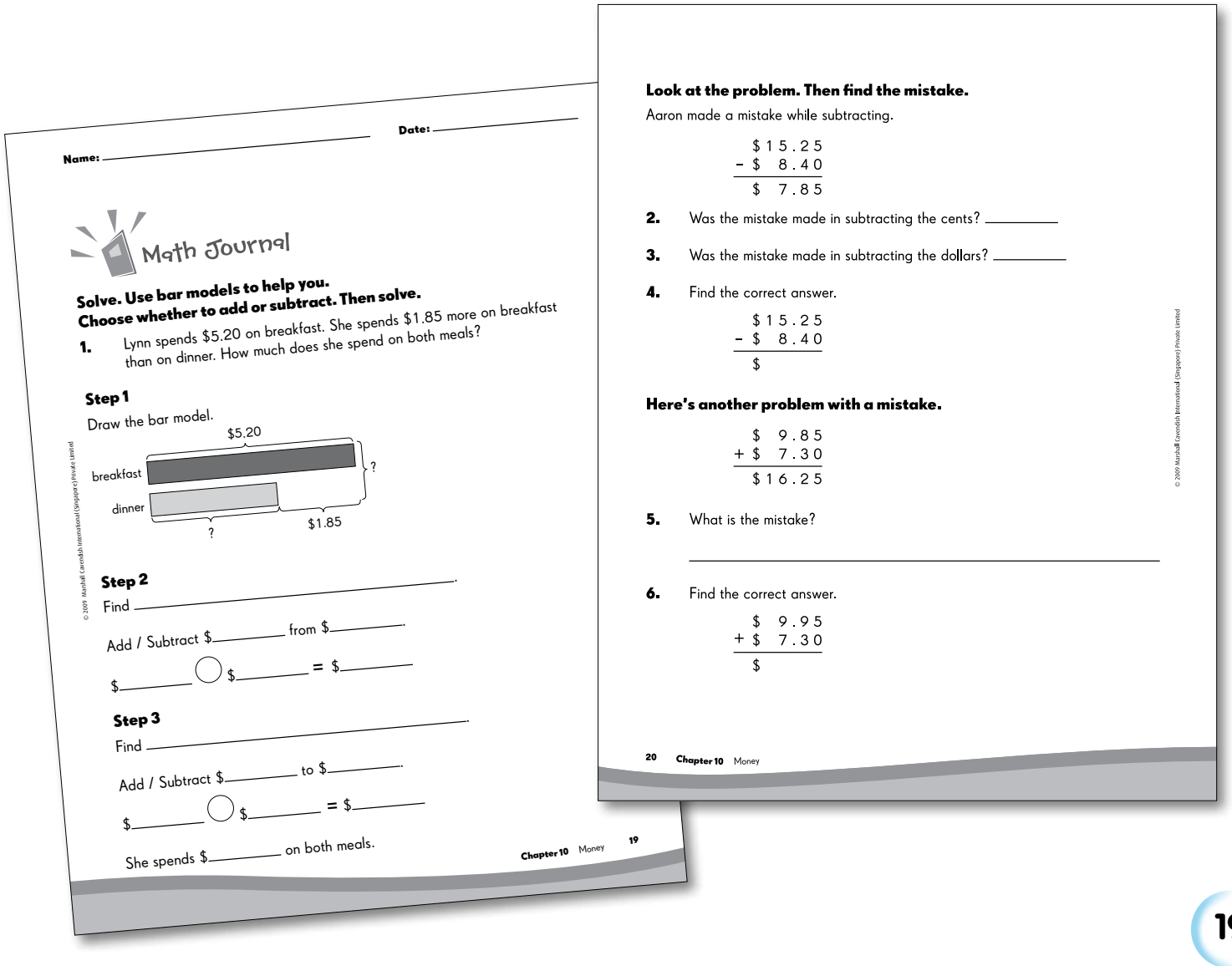
Teaching Metacognition

According to the Curriculum Planning and Development Division (CPDD) of the Ministry of Singapore, metacognition, or "thinking about thinking," refers to the awareness of, and the ability to control one's thinking processes, in particular the selection and use of problem solving strategies. It includes monitoring one's own thinking and self-regulating learning.

"(C)ognitive science studies of problem solving have documented the importance of adaptive expertise and of what is called *metacognition:* knowledge about one's own thinking and ability to monitor one's own understanding and problem solving activity" (NATIONAL RESEARCH COUNCIL, 2001, P. 118).

Metacognitive awareness can be developed through:

- Exposure to general problem solving skills, thinking skills, and heuristics, and how these skills can be applied to solve problems.
- Modeling of strategies and methods for solving problems, as through think-alouds.
- Problems that require planning (before solving) and evaluation (after solving).
- Encouragement to seek alternative ways of solving problems.
- Requiring students to check the appropriateness and reasonableness of answers.
- Allowing students to discuss how to solve a particular problem and to explain the different methods for solving the problem.



Name: _____ Date: _____

Math Journal

Solve. Use bar models to help you.
Choose whether to add or subtract. Then solve.

1. Lynn spends $5.20 on breakfast. She spends $1.85 more on breakfast than on dinner. How much does she spend on both meals?

Step 1

Draw the bar model.

$5.20
breakfast
dinner
?
?
$1.85

Step 2

Find _____.

Add / Subtract $_____ from $_____.

$_____ ◯ $_____ = $_____

Step 3

Find _____.

Add / Subtract $_____ to $_____.

$_____ ◯ $_____ = $_____

She spends $_____ on both meals.

Look at the problem. Then find the mistake.

Aaron made a mistake while subtracting.

$15.25
− $ 8.40
$ 7.85

2. Was the mistake made in subtracting the cents? _____

3. Was the mistake made in subtracting the dollars? _____

4. Find the correct answer.

$15.25
− $ 8.40
$

Here's another problem with a mistake.

$ 9.85
+ $ 7.30
$16.25

5. What is the mistake?

_____

6. Find the correct answer.

$ 9.95
+ $ 7.30
$

PRINCIPLES FROM RESEARCH

"Teaching practices congruent with a metacognitive approach to learning include those that focus on sense-making, self-assessment, and reflection on what worked and what needs improving. These practices have been shown to increase the degree to which students transfer their learning to new settings and events" (BRANSFORD, ET AL., 2000, P. 12).

"Metacognition and adaptive reasoning both describe the phenomenon of ongoing sense making, reflection, and explanation to oneself and others" (DONOVAN & BRANSFORD, 2005, P. 218).

Students' metacognition helps them to apply learning to new contexts: "By *describing their own processes,* students can use their reflections to develop flexible prototypes of experiences that can be drawn on in future problem solving" (LESH & ZAWOJEWSKI, 2007, P.770).

TURNING PRINCIPLES INTO SUCCESS

The metacognitive approach is foundational to the ***Math in Focus:*** **The Singapore Approach** program. The concrete to pictorial to abstract sequence provides the structure necessary for students to grasp the "how and why" of mathematical concepts. Discussion is encouraged through Hands-On activities and investigation and reasoning skills are enhanced through the Let's Explore! and Put On Your Thinking Cap! activities.

Math in Focus: **The Singapore Approach** has embedded communication and reflection opportunities throughout the program so that students are able to discuss their thinking and reflect on the math they are practicing. The Math Journal is one of the ways that students are able to reflect on their own learning.

Fostering Student Motivation/Improving Student Attitudes

Motivation describes the internal process that allows a person to initiate, persist, and complete activities. Increasing student motivation and a positive attitude towards learning determines how well students learn. Gaining a strong foundation in mathematics concepts develops students' positive attitudes about math and their confidence in their future success. Indeed, the National Research Council (2001) reports: "Students' disposition toward mathematics is a major factor in determining their educational success" (P. 131). The sequence of learning, practice time, use of models, and ongoing diagnostics in ***Math in Focus:*** **The Singapore Approach** cultivate a student-centered learning environment that fosters student motivation and improves student attitudes about learning mathematics.

PRINCIPLES FROM RESEARCH

In mathematics, proficiency is linked to motivation and attitudes: "In addition to the concepts and skills that underlie mathematical proficiency, children who are successful in mathematics have a set of attitudes and beliefs that support their learning. They see mathematics as a meaningful, interesting, and worthwhile activity; believe that they are capable of learning it; and are motivated to put in the effort required to learn" (NATIONAL RESEARCH COUNCIL, 2001, P. 171). For this reason, teaching students the strategies to be proficient in mathematics is essential: "Because it leads to better performance, strategy use raises self-efficacy and motivation. Use of strategies relates positively to achievement and self-efficacy…" (SCHUNK, ET AL., 2008, P. 310).

According to the National Research Council (2001), "as students build strategic competence in solving nonroutine problems, their attitudes and beliefs about themselves as mathematics learners become more positive. The more mathematical concepts they understand, the more sensible mathematics becomes… Similarly, when students see themselves as capable of learning mathematics and using it to solve problems, they become able to develop further their procedural fluency or their adaptive reasoning abilities" (P. 131).

TURNING PRINCIPLES INTO SUCCESS

Math in Focus: **The Singapore Approach** weaves problem solving throughout each lesson. The problems grow in difficulty from one-step, to two-step, to multi-step, to non-routine. Students are encouraged to discuss and approach problems using unique approaches (Let's Explore! and Put On Your Thinking Cap!), but are taught a systematic process for tackling problems using bar models so that they know where to start and have a positive attitude about word problems in general and a confidence in their own abilities to succeed.

With each new chapter in ***Math in Focus:*** **The Singapore Approach,** teachers are provided with activities to help students recall prior knowledge necessary for new concepts and skills. Common Error alerts for identifying and correcting common misconceptions and Best Practice tips for teaching the lesson and classroom management are also provided so that the teacher can provide the support necessary for students to succeed. By building on previous successes, the teacher provides students with opportunities to build their confidence with new concepts. The multi-models used to present the new concepts help to give students the confidence and motivation to complete the two-step and multi-step problems presented in the Hands-On activity and the Let's Explore! investigative activities.

Conclusion

Singapore has placed at the top in the last four administrations of the Trends in International Math and Science Study (TIMSS); the Singapore mathematics program is largely believed to be responsible for this exceptional performance. ***Math in Focus:*** **The Singapore Approach** addresses those key evidence-based features in the Singapore mathematics program that have made the program so successful, and presents these features in a version of the program specifically designed to meet the needs of American students and educators.

Research in cognition generally and in mathematics education specifically supports the many features which characterize ***Math in Focus:*** **The Singapore Approach.** The program employs the strategic development of concepts throughout the current grade and successive grades to allow for a deeper understanding of the skills, without unnecessary repetition. Extensive practice opportunities allow for skill review both before and during lessons, and for the applications of concepts and skills to new contexts. Deliberate scaffolding supports the development of students' reasoning skills as they work on successively more intricate math problems and move from solving simple routine problems to independently solving complex, non-routine word problems.

By using evidence-based teaching and learning strategies and a curriculum model demonstrated by research to be effective, ***Math in Focus:*** **The Singapore Approach** prepares students for future learning in mathematics and develops the problem solving and critical thinking skills needed in the 21st century.

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