Math Handbooks Problem Solving:
Developing Thinking Skills Through Reading and Writing

Problem solving is an integral part of all mathematical learning.

The National Council of Teachers of Mathematics has identified problem solving as one of the five fundamental mathematical process standards along with reasoning and proof, communication, connections, and representations (National Council of Teachers of Mathematics [NCTM], 2000). "Problem solving is the foundation of all mathematical activity" (Reys, Lindquist, Lambdin, Smith, & Suydam, 2001). "In order to function in our complex and changing society, people need to be able to solve a wide variety of problems. The elementary math curriculum must prepare children to become effective problem solvers" (Burns, 2000). Problem solving shouldn't be a separate process, but rather the context within which students learn math skills and concepts (Zemelman, Daniels, & Hyde, 1998). Although problem solving is an integral part of all mathematics, many students struggle with solving problems. In fact, students' "ability to solve word problems falls far below their ability to compute" (Burns, 2000). Research shows that this discrepancy is not because children have poor computation or reading skills, but because children "do not know how to choose the correct operation to apply to the problem" (Burns, 2000). With explicit instruction in problem solving students can and will improve.

The *Problem Solving* books help students develop the mathematics, reading, writing, and thinking skills they need to solve math problems typically found in textbooks and on standardized tests and to transfer these skills to real-life math scenarios. "Problem solving ability is enhanced when students have opportunities to solve problems themselves and to see problems being solved. Further, problem solving can provide the site for learning new concepts and for practicing learned skills. We believe problem solving is vital because it calls on all strands of proficiency, thus increasing the chances of students integrating them" (Kilpatrick, Swafford, & Findell, 2001). The *Problem Solving* books offer a scaffolded problem-solving plan to help students understand and apply problem-solving strategies to a variety of problems.

"Problem-solving ability does not develop over a few weeks or months. Nor is it a topic that is taught at a particular grade level . . . . We need to address problem solving virtually every day, in every lesson, beginning in kindergarten and continuing through high school, because problem solving and learning mathematics are so intimately connected" (Van de Walle, 1994). The *Problem Solving* books provide daily opportunities to practice a variety of skills and strategies such as checking and revising, reading diagrams, graphs, tables and charts, writing equations, and using logical reasoning. Real-world opportunities to solve problems allow students to see the usefulness of mathematics. The *Problem Solving* books present the mathematics in context, engage students, and promote flexible thinking.

Direct instruction of a problem-solving strategy improves student performance on problem solving.

The problem-solving approach taught in the *Problem Solving* books is based on the work of George Polya. His problem-solving model involves four stages: understand the problem, devise a plan for solving the problem, carry out your plan, and look back to examine the solution obtained (Reys, et al., 2001). Meta-analysis of 487 studies on problem solving indicates that students trained to use a heuristic model such as Polya's model showed the largest gains in problem-solving performance. Students must learn to apply and adapt a variety of appropriate strategies to solve problems. These "strategies include using diagrams, looking for patterns, listing all possibilities, trying special values or cases, working backward, guessing and checking, creating an equivalent problem, and creating a simpler problem" (NCTM, 2000). The *Problem Solving* books address these strategies by providing students with the opportunity to solve problems in more than one way, find needed information, ignore unneeded information, make or read diagrams, tables or graphs, write equations, solve multi-step problems, use logical reasoning, create an organized list, and combine strategies. The *Problem Solving* books include charts, graphs, and other tools that organize and display data. Students learn how to find, evaluate, and use all sorts of information. In the process they practice reasoning and critical thinking skills.

The *Problem Solving* books provide a step-by-step approach that helps students learn and practice all four parts of the problem solving method—understand, plan, try, and look back. The problem-solving step taught in each chapter is highlighted across the top of the page to help keep students focused. As students learn to use the four-part problem solving strategy they also develop the specific skills such as diagramming that they need to solve problems. Instruction in diagram drawing and translating words into mathematical symbols indicate the greatest positive effects toward better performance in solving problems (Hembree, 1992; NCTM, 2000). These sub-skills are important in representing problems, which is the foundation for understanding.

Teaching students strategies for reading mathematical texts helps them improve both their mathematics ability and their literacy skills.

"Reading, writing, and mathematics are, or should be, inseparable" (Sutton & Krueger, 2002). The Commission on Reading suggests that most reading and thinking strategies should be taught in the content areas, rather than isolated reading lessons (Anderson, Hiebert, Scott, & Wilkinson, 1984). Research suggests that content-area reading comprehension is critical to student success in all disciplines and at all levels of competency. "A thorough understanding of language and literacy is essential for learning in any subject, because it makes the world more understandable, more interesting, and more connected to daily life" (Hamm & Adams, 1998). When taught together, reading and mathematics process skills can reinforce one another.

"Many of the process skills needed for mathematics are similar to reading skills, and when taught together would reinforce each other" (Sutton & Krueger, 2002). These common skills include activating prior knowledge, predicting, synthesizing information, making inferences, drawing conclusions, asking questions, making connections, comparing and contrasting, evaluating resources, communicating, and building vocabulary. In the *Problem Solving* books, these reading and mathematics skills are interwoven as students learn to apply them to improve both their reading and mathematics comprehension.

The *Problem Solving* books promote critical reading and thinking skills in the context of engaging lessons focusing on key mathematical concepts. The *Problem Solving* books adapt techniques used in reading instruction to teach students how to picture what is happening in a math problem, to develop a plan, and then to solve and check their work. Students are asked to read critically about a variety of cross-curricular topics and solve math problems related to those topics. With the right tools to support their knowledge, students meet success as readers and problem solvers.

Writing about mathematics is an important tool for building students' mathematical comprehension.

Incorporating writing into problem solving has long-term positive effects. Many of the process skills used in mathematics are also used in writing (i.e., finding information and interpreting data). Writing in mathematics can help students organize their understanding of mathematical ideas (NCTM, 2000) and solving problems and defining mathematical terms in one's own words can increase students' writing competence (Sutton & Krueger, 2002). "Writing is not simply a way for students to demonstrate what they know. It is a way to help them understand what they know. At its best, writing is learning" (National Commission on Writing in America's Schools and Colleges, 2003). In addition, "talking and writing about mathematics are essential parts of learning mathematics" (Reys, et al., 2001). A report issued by The National Commission on Writing in America's Schools and Colleges emphasizes the importance of writing as a learning tool in all subject areas and urges state and local education agencies to greatly increase the amount of time students spend writing by encouraging writing in all subject areas (National Commission on Writing in America's Schools and Colleges, 2003). The *Problem Solving* books help schools incorporate writing into their mathematics classes.

A wide range of exercises that support the kinds of writing students are required to do in mathematics are included in the *Problem Solving* books such as paraphrasing, summarizing, generating lists, drawing and labeling diagrams, creating charts and tables, comparing and contrasting, making inferences, and interpreting data. Students' ability to write about mathematics also provides teachers with an opportunity to assess what they have learned. The *Problem Solving* books provide students with the opportunity to write about the mathematics they are learning and help educators promote writing across the curriculum.

The *Problem Solving* books also help promote students' vocabulary development. Vocabulary instruction is often overlooked in mathematics instruction and its inclusion offers a point of entry for students who have difficulty with computation (McKeown & Beck, 1988). "[V]ocabulary is a crucial part of instruction in content areas" (Graves & Slater, 1996). Math vocabulary within each lesson of the *Problem Solving* books is identified and defined at point of use. Students also have the opportunity to define vocabulary words using their own words, drawings, and examples to create their own personal math glossary. This writing activity helps students organize, clarify, and refine their understanding of important math-related vocabulary.

An integrated curriculum offers students a meaningful context for learning.

"Interconnections among the disciplines . . . will support learning by making the mathematics curriculum more meaningful" (Sutton & Krueger, 2002). Research has shown that true learning occurs when the information being taught makes sense to and has meaning for the student (Sutton & Krueger, 2002). In addition, the "separate-subject approach too often leaves students with a disconnected view of knowledge and fails to reflect the way that real people attack real problems in the real world. In school, knowledge and learning are typically compartmentalized, offering no view of how it all comes together to reveal the big picture" (Daniels & Bizar, 1998). "Children who cannot apply their reading, writing, and math skills to real-world situations are not being educated to succeed beyond the walls of their classrooms" (Cunningham & Allington, 1999). Integrating math instruction with reading, writing, science, social studies, and history not only helps students apply their problem-solving abilities across the curriculum, but also provides students with an alternative point of entry.

In the *Problem Solving* books, each chapter focuses on a different cross-curricular theme from science, social studies, history, and geography ranging from the wonders of Egypt and the Underground Railroad to volcanoes and insects. High-interest thematic reading selections provide a real-world context for practicing the mathematical skills and strategies needed to solve problems. In addition, the integration of cross-curricular topics provides students who struggle with mathematics with another way to approach difficult concepts. "If the goal is to produce mathematically literate citizens who can apply mathematical thinking in real-life problem solving, then subject integration is essential" (Sutton & Krueger, 2002). Students who enjoy other subject areas are engaged by the content that helps them to become more interested in mathematics and more successful.

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